THC White Paper Series #5
Low Interest Rate Regime: Challenges and Solutions

MARKET INTEREST RATE FORECAST (MIRF)

AN ARBITRAGE-FREE INTEREST RATE MODELING APPROACH

by
Thomas Ho, Ph.D.
CEO / Founder

January, 2020

Keywords:

rate forecast, rate distribution, forward rates, distribution skewness, arbitrage-free models, Monte Carlo Simulation, Binomial Lattice Models, Finite Difference, ATM OTM Calibration

TABLE OF CONTENTS

PROLOGUE

I described the limitations of some of the current operational interest rate models in today's low rate regime in my last paper. I show that these models accept neither negative interest rates nor negative rate distribution skewness, and therefore, these models can misprice options when rates are low.

I have also explained the principle of an arbitrage-free interest rate model, which is to infer the interest rate movements from the capital market, instead of basing on subjective interest rate forecast in pricing fixed-income instruments. THC White Paper [4] shows that many operating interest rate models fail to incorporate full capital market information, resulting also in the mispricing of options.

The capital market's perception of rate movements is priced in the at-the-money and out-of-the-money options. Financial models can use an arbitrage-free interest rate model based on capital market prices to forecast yield curve movements based on the expected value and the critical values. The Arbitrage-Free interest rate modeling approach to rate forecasts, including stress scenarios, is objective, systematic and transparent.

This paper continues to explore important applications of an arbitrage-free interest rate model

INTRODUCTION

Interest rate forecasting is central to the ALCO meeting. Rate forecasting is not confined to the expected yield curve over a time horizon, such as three months or one year. Depending on the application, ALCO decisions may also depend on the rise of interest rates for a specified confidence level.

One key economic value of a developed capital market is price discovery. The market aggregates the participants' views to determine the yield curve, at-the-money, and out-of-the-money option prices. While rate forecasters provide their subjective views of the market, the capital market provides the market consensus. If the market consensus is inappropriate, market participants will arbitrage, resulting in the market prices reflecting market consensus. The arbitrage-free interest rate model seeks to report such market consensus of future movements of the yield curve.

"Pricing of Interest Rate Contingent Claims" Ho-Lee introduces the Local Volatility Model of an arbitrage-free interest rate model in 1984. Because of the categorical rejection of any possibility of negative interest rates occurring at the time, the concept of estimating volatilities as each node of a recombining binomial lattice was abandoned. Instead, lognormal models have become a standard approach. THC White Paper [4] has noted that these models prespecify the market Rate Distribution and therefore, cannot fully infer the yield curve movement from the capital market prices.

By way of contrast, THC White Paper [1] introduces the Local Volatility Model that can present the Market Rate Distributions. The purpose of this paper is to present the applications of the Local Volatility Model in interest rate forecasting, providing empirical evidence and applications. The mathematical derivation of the model is provided in the Technical Notes.

MONTE-CARLO SIMULATIONS AND RECOMBINING LATTICE

One common approach to generating arbitrage-free interest rate projections is the Monte-Carlo methodology. The Monte-Carlo methodology's basic framework is to derive an arbitrage-free interest rate model in the form of differential equations. The Monte-Carlo Model projects hundreds of random interest rates, which can determine the value of a fixed-income instrument. This approach has several limitations in today's environment, as explained in the THC White Paper [4].

In particular, THC White Paper [4] presents a set of random arbitrage-free interest rate paths, Monte-Carlo Simulations.

The 257 paths used in Figure 1 represent only a tiny fraction of all the possible 100 100 scenarios.

Figure 1 seems to suggest that the simulations of 257 interest rate paths cover many future scenarios. But such is not the case when compared with the sample size space required to value fixed income instruments in general. To determine when an option is to be called or put depends on many factors, such as the history of the interest rate paths, the expected future interest rates, and the deviations from the expected interest rates.

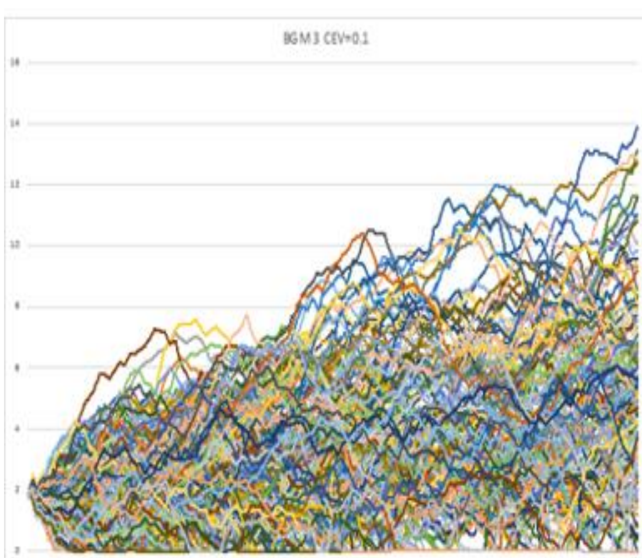


Figure 1.

MONTE-CARLO SIMULATIONS AND RECOMBINING LATTICE

A mathematical tool, called Finite Difference, is used to model all the future states precisely. There is a minimum of 2^{360} (approximately 10^{100}) possible interest rate paths to determine option value. Therefore, the 257 paths only represent a tiny fraction of all the possible scenarios.

There are two limitations to the Monte-Carlo Method. First, a comprehensive description of the paths is necessary to determine the Rate Distribution, and a few hundred interest rate paths cannot construct an accurate Rate Distribution

Second, many interest rate models fail to be arbitrage-free because the approach typically cannot incorporate a broader range of calibration instruments because they often cannot fit the ATM and OTM options.

MARKET INTEREST RATE FORECAST (MIRF)

An arbitrage-free rate model is a normative theory, and hence powerful for capital market applications

The Local Volatility Model can recover the Rate Distribution from the capital market prices to determine the Market Interest Rate Forecast (MIRF). ALCO, market risk management, portfolio services, and treasury can customize the Rate Distributions for their specific applications. MIRF can be customized to the needs:

- the change in the expected yield curve over a specific horizon for positioning the balance sheet or portfolio;
- a particular rate over a time horizon to determine the short-term funding rates or the fixed-rate mortgage rate with a margin of a specific benchmark rate;
- the stressed rates for a specific confidence level for capital management, haircut calculation for collateral in lending, such as the rates with a 90% confidence level, the market rates will not exceed or fall below.
- Rates for balance sheet or portfolio total return strategies under alternative probabilities of rate shifts; this approach differs from assuming parallel shifts of the yield curve such as 100, 200, 300 bp shifts, as commonly used.

An arbitrage-free rate model belongs to the class of normative theories. MIRF presents a value that is "ought to be" based on the arbitrage activities in the capital market. If the market is efficient, then the expected values truly reflect the market consensus of future rates. If the expected rates are inappropriate, then, the model suggests arbitrage opportunities may be available, barring frictions in transactions.

COMPARE MIRF AND EXPECTATION HYPOTHESIS

THC White Paper [4] explains the methodology of using the Local Volatility Model to determine the one-month swap rate distribution over any time horizon. Because the Rate Distribution is skewed, the mean of the distribution should differ from the median, which is approximately the forward rate.

The Expectation Hypothesis suggests erroneously that the forward curve is the expected yield curve because the forward curve aggregates all market participants' expectations of rate forecasts to determine the shape of the yield curve. If investors believe that rates will go up, then the yield curve would be upward sloping. When market participants expect a recession is imminent, the yield curve would be downward sloping.

Figure 2 below shows that the Rate Distribution is not symmetrical, with the variance and skewness changing continually over time. The skewness affect the expected interest rate levels

However, the Expectation Hypothesis forecast is only correct if there is no interest rate risk. When interest rate risk is present, the rate distribution can be either positively or negatively skewed. As a result, the average rate of the rate distribution, as forecasted by the capital market, is not the same as the forward rates. The results below illustrate the difference between the forward rates and the forecasted rates.

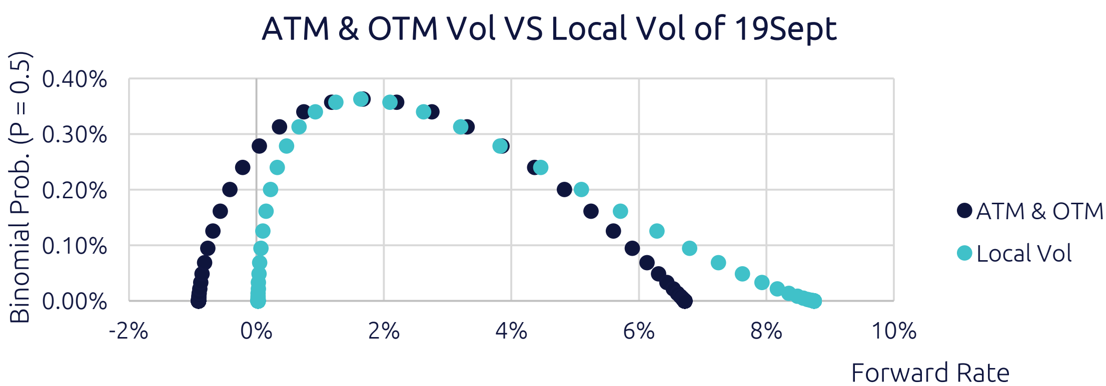


Figure 2.

INTEREST RATE FORECAST AND FORWARD RATES COMPARISON

Over the sample period, the swap curve is downward sloping from the 3-month term to the five-year term

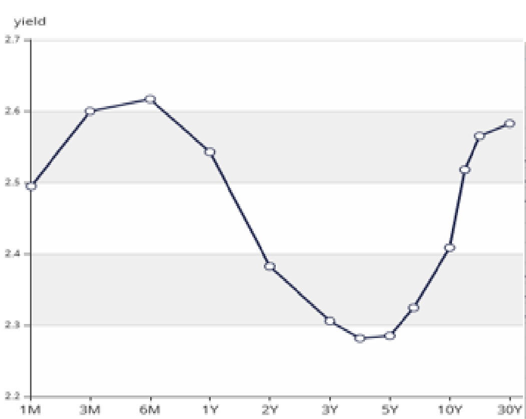
This section presents results comparing the expected one-month swap rate with the forward ten year, five year and three-year rate, over the sample period from 3/30/2019 – 11/30/2019. The results show that when the interest rates are low and uncertainty high, the rate distribution is positively skewed as depicted by the graphs. These yield curves generate the forward rates that will benchmark the Rate Forecast.

I present a comparison of the Market Forecasted Rates and the forward rates below. The empirical evidence uses ATM and OTM swaptions to calibrate the Local Volatility Model, which allows for the capital market to determine the skewness of the rate distribution.

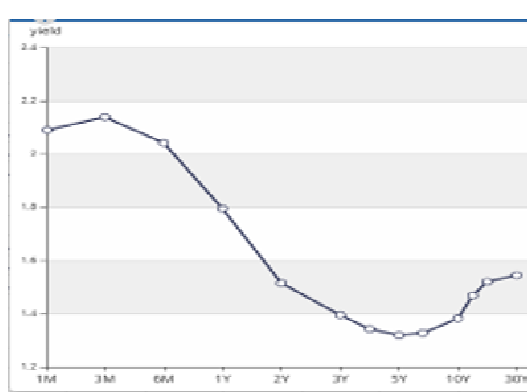
Over the first period, the swap curve is downward sloping from the 3-month term to the five-year term.

In the next period, the rates fell. The five-year rate fell from 2.28% to 1.32%, while the swap curve remained downward sloping.

3/30/2019



8/30/2019



INTEREST RATE FORECAST AND FORWARD RATES COMPARISON

The results show that:

- While the expected rates are highly correlated with the forward rates, the difference can be significant and dynamic, changing monthly over the sample period.

- The one-month rate is expected to be 0.48% higher than the corresponding forward rate over a 10-year horizon in August 2019, showing that the expected rate did not fall in tandem with the forward rate. The forward rate dropped from 2.30% to 1.60% while the expected rate dropped from 2.51% to 2.08%

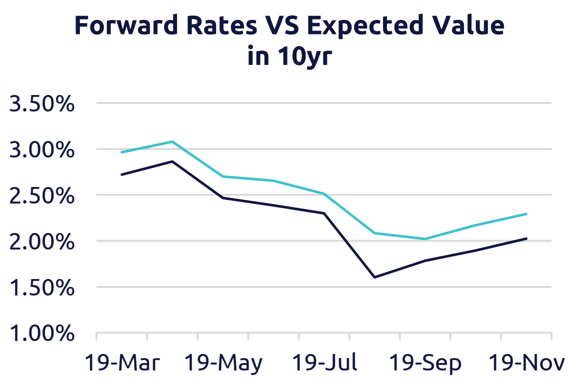
- Since the forward rates and the expected rates must converge to the spot rate as the time horizon approaches the daily rate, the results below show that the spread between the expected rate shrinks with the forward rates. But the shrinking of the 3, 5, 10-year spread did not move in tandem over time. For example, the five-year spread became tight in November, but the 10-year spread remained wide.

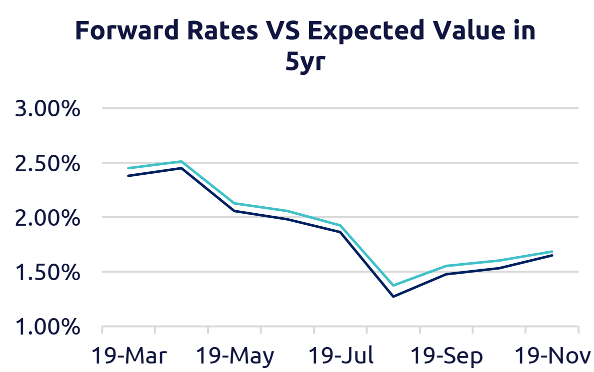
Empirical Evidence

These are some examples of the applications of MIRF.

- For illustrative purposes, I present below the projected one-month rates on 3/30/2019, 6/30/2019 and 9/30/2019. The results show that by 9/30/2019, the market expected the short-term rates to rise to 2.13% and fall to 1.75% by December 2019 and March 2020, respectively from the spot rate of 2.04%. As of November 2019, the market FED rate is 1.56%, with a target of 1.75%.
- The short-term rate is projected to fall to 1.39% in 12 months, before starting to rise to 2% in 10 years. The results show the dynamic nature of the expected rates. From March to September, the expected 10- year rate has dropped 70 bp.

	19-Mar	19-Apr	19-May	19-Jun	19-Jul	19-Aug	19-Sep	19-Oct	19-Nov
Time	10 year	10 year	10 year	10 year	10 year	10 year	10 year	10 year	10 year
Forward Rates	2.72%	2.86%	2.47%	2.39%	2.30%	1.60%	1.78%	1.90%	2.03%
Expected Value	2.96%	3.08%	2.70%	2.66%	2.51%	2.08%	2.02%	2.17%	2.29%
Time	5 year	5 year	5 year	5 year	5 year	5 year	5 year	5 year	5 year
Forward Rates	2.38%	2.45%	2.06%	1.98%	1.86%	1.27%	1.48%	1.53%	1.65%
Expected Value	2.45%	2.51%	2.13%	2.06%	1.92%	1.37%	1.55%	1.60%	1.69%
Time	3 year	3 year	3 year	3 year	3 year	3 year	3 year	3 year	3 year
Forward Rates	2.17%	2.22%	1.82%	1.68%	1.69%	1.16%	1.39%	1.41%	1.53%
Expected Value	2.19%	2.24%	1.84%	1.70%	1.71%	1.19%	1.41%	1.43%	1.54%





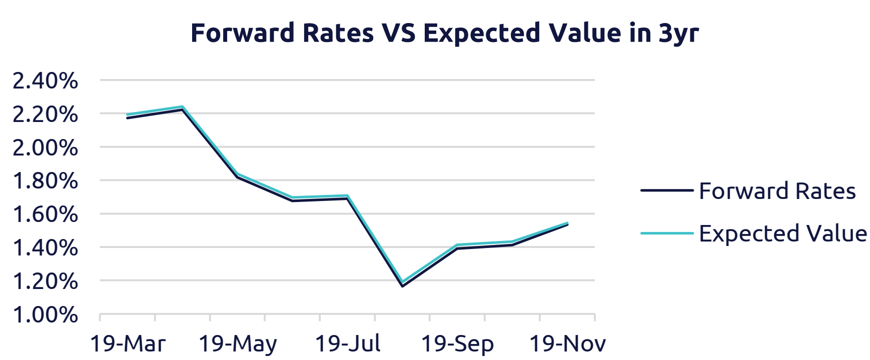


Figure 4.

APPLICATIONS

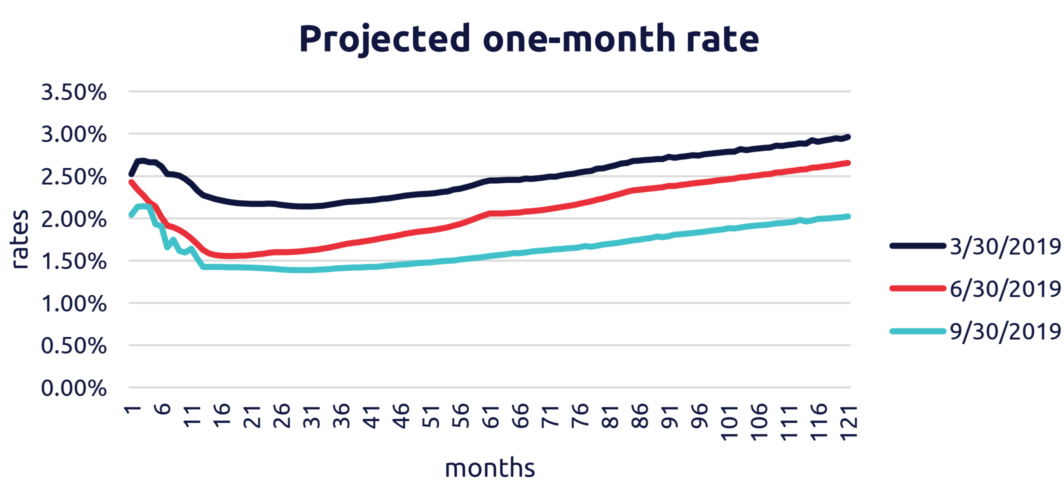
MIRF can be used for

- *ALCO meeting*
- *Budgeting*
- *Total return analysis*
- *DFAST and Stress Test*
- *Benchmark/Indices for Loan Pricing*
- *Balance Sheet duration exposure*

Application 1.

MIRF differs from typical interest rate forecasting offered by consulting firms for the following reasons:

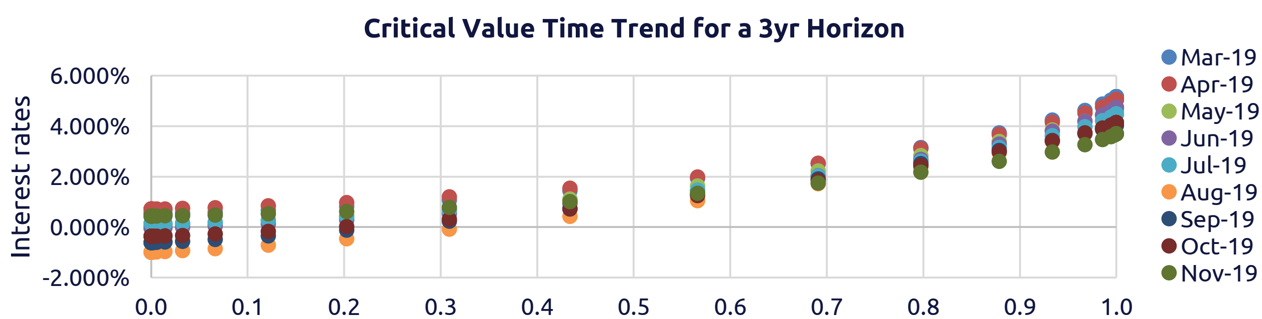
- the rate forecast is objectively based on capital market prices of fixed income instruments and derivatives;
- the rate distributions can be customized in multiple ways, particularly for risk management, stress testing, and total return analysis, and are not confined to rates rising or falling;
- the rate is consistent with capital market pricing of fixed income instruments, and therefore, the normative nature of the arbitrage-free model relates the expected rates and capital market pricing of fixed-income instruments and derivatives, thus providing transparency of profitable transactional opportunities.

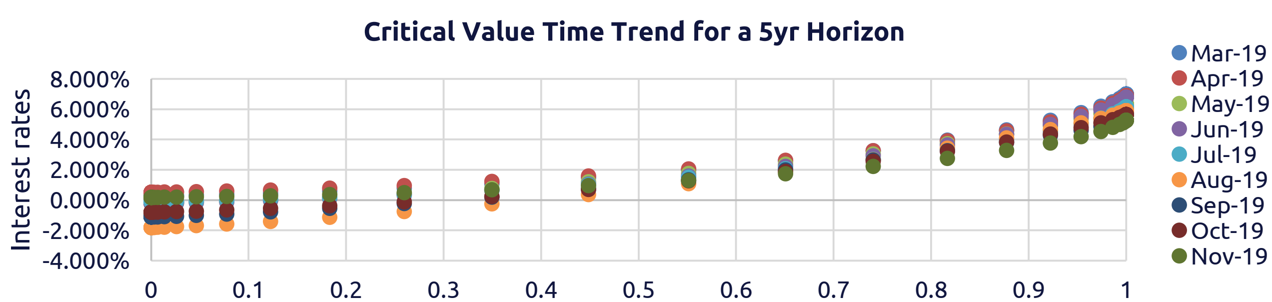


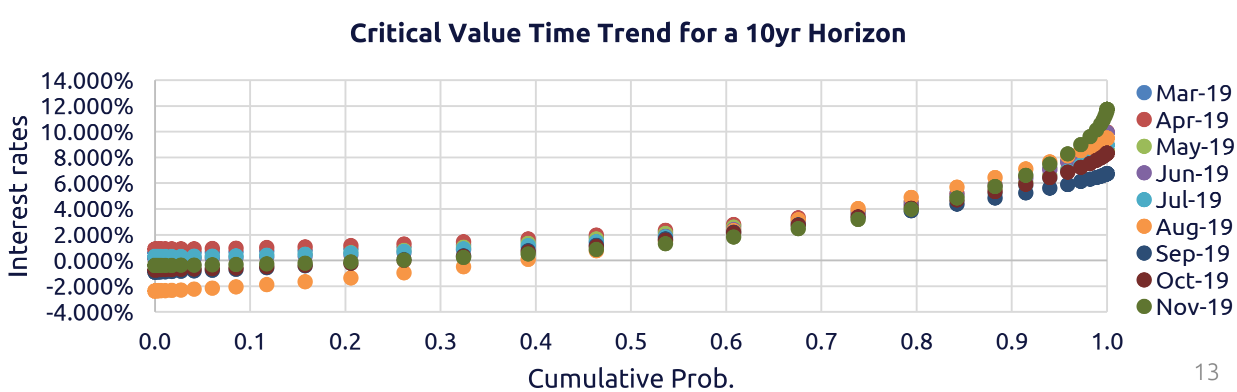
Application 1.

Application 2.

The plot of rates against the cumulative probabilities presents the maximum or minimum rate for a one tail probability. The results shows that the critical value depends on the expected rate, the rate volatility and skewness. The expected rate, volatility and skewed are represented by the level at 0.5, slope and curvature of the plots. The results show that the three attributes change dynamically







CONCLUSIONS

Free of the constraints that arbitrage-free interest rates have to be positive, MIRF can provide many applications

Recent years have proven that negative interest rates are possible. The prevalence of negative interest rates contradicts the long-held dogma that interest rates must be positive and has shaken the fundamental assumptions of many currently operational interest rate models.

Without the constraint on interest rate models generating negative interest rates, I propose a more general class of interest rate models that can use a portfolio of broader range of options to calibrate the interest rate model. As a result, the interest rate model can capture the market forecasted rates more accurately and price fixed-income instruments and derivatives accurately, following the normative nature of an arbitrage-free interest rate model.

Interest rate models are the bedrock to formulate the credit model because interest rate models are the only models that can model the life-of-loans under alternative interest rate scenarios and stress tests. The introduction of CECL in financial reporting underscores the importance of the results reported in this paper. Enterprise Risk Management has to report balance sheet fair value and simulated incomes consistently. The methodology presented in this paper provides a consistent modeling framework.

The Local Volatility Model remains applicable in the low rate environment. Furthermore, by capturing the skewness of the Rate Distribution, MIRF can provide valuable rate benchmarking indices and risk management.

EPILOGUE

The realization of the possibility of negative interest rates has challenged our current thinking of interest rate modeling

Currently, the implementation of interest rate models, such as MIRF, provide more accurate pricing of options and capital market inference of rate distributions

Interest rate models are central to enterprise risk management, trading, portfolio management, and more. They form the bedrock of financial engineering. One has to wonder why so many interest rate models have such significant limitations, despite the significant progress made in research in the past 35 years of financial modeling.

I believe such an apparent paradox comes from a historical accident. The first arbitrage-free interest rate model [1984] was a normal model, introducing the local volatilities concept. But at the time, the possibility of negative interest rates was categorically rejected and has become a dogma. Also, at the time, economic interest rate modeling that relates to the real economy (Cox-Ingersoll-Ross (1979)) was using a lognormal model. Lognormal models (or normal models with negative rates floored) were then adopted over the following 35 years. Financial engineers use mathematical tools to solve the Monte-Carlo approach of the LIBOR Market Model. The low-interest rate scenarios challenge financial modelers to evaluate the fundamental normative nature of arbitrage-free interest rate modeling. It is important to note that financial economics theory lead market participants to actionable decisions while mathematics provides commutation efficiency. Mathematics alone cannot provide many business solutions.

Therefore, this series of five papers (see reference) covers a topic of central importance to market making, portfolio management, and enterprise risk management.

White Paper [1] introduces an interest rate model appropriate for the current low-interest-rate regime (the mathematical model is presented in the Technical Notes).

EPILOGUE

White Paper [2] explains the importance of measuring option value accurately to manage profits, showing that the traditional accounting approach can be erroneous when identifying loan profitability.

White Paper [3] describes the pricing of embedded options, which are prevalent on the balance sheet, underscoring the importance of using an appropriate interest rate model in a low-interest rate regime.

White Paper [4] provides a comparison and contrast of the current interest rate models, highlighting the limitations of interest rate models that ignore the possibility of negative interest rates. Finally,

White Paper [5], this paper, uses the Local Volatility Model to introduce Market Interest Rate Forecast (MIRF), which has many significant applications to banking processes.

.

TECHNICAL NOTES

The Local Volatility Model lets the capital market Fund Transfer Pricing (FTP) rates, the At-the-Money and Out-of-the-Money determine the interest rate model, which takes the form:

$dr = \emptyset(t)dt + \sigma(t)\eta(r)dz$ (1)

where

dr small change of a short-term rate,

$\emptyset(t)$ an adjustment term to ensure the model is calibrated to the capital market prices,

$\sigma(t)$ the term structure of volatilities

$\eta(r)$ probability rate distribution

dz small random uncertain movement

The Local Volatility Model uses a finite difference methodology. In particular, the approach uses the recombining binomial lattice model to represent equation 1. The one period discounted value P_i^n at time n and state i is derived below (White Paper [1] Ho-Mudavanhu)

$$P_i^n = \frac{P(n+1)}{P(n)} \prod_{k=1}^{i} \frac{1+\delta_0^{k-1}(i-k)}{1+\delta_0^{k-1}(i-k+1)} \prod_{m=0}^{i-1} \delta_m^{n-1}$$

$$\mathrm{\delta_i^n = \exp(-2f(n). n . g(i, n; p)\sqrt{\Delta t})}$$

Where

$\mathrm{f(n) = (a + bn) \exp(-c\, n) + d}$, modeling the volatility depending on the term n

g(i, n; p) a discrete probability distribution function over states i at time n where p is the distribution parameter

For any time n, the rate at time n and state i, r (i, n), as the risk-neutral probability of b(i, n; 0.5), the normal, binomial distribution, number of rate paths that reach the node at time n and state i.

The Rate Distribution at time n is defined as the scattered plot D(r) = (b(i, n; 0.5), r (i, n)) over indices i.

The rate frequency distribution derives the mean (expected) value of rates, the base case and shocked yield curves over any time horizons,

REFERENCES

1. THC White Paper October 2019 "Arbitrage-Free Interest Rate Model in Negative Rate Regimes"

2. THC White Paper November 2019 "pOAS: a New Loan Profitability Measure."

3. THC White Paper December 2019 "Managing Option Risk using Local Volatilities."

4. THC White Paper November 2019 "Interest Rate Models: Re-Examine Option Pricing"

5. THC White Paper December 2019 "Rate Forecast: An Arbitrage-Free Interest Rate Model Approach"

About the Author

Thomas S. Y. Ho. Ph.D., is CEO and Founder of Thomas Ho Company Ltd., a New York-based financial engineering company. He is the co-author of the Ho-Lee Model and has over 35 years of experience in financial modeling and is the author of more than 80 books and refereed publications.

THC Financial Engineering

Please contact us:
customerservice@thomasho.com
www. thomasho.com
phone: (269) 558-5004

Thomas Ho Company, Ltd.
New York Office:
55 Liberty St, Suite 4B
New York, NY 10005

Michigan Office:
14900 West Michigan Ave.
Marshall, MI 49068